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1 The influence of heterogeneity on the strength of volcanic rocks and the stability
2 of lava domes

3

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26

27 **Abstract**

28 The collapse of lava domes, inherently heterogeneous structures, represents a significant volcanic hazard.

29 Numerical and analogue models designed to model dome instability and collapse have incorporated heterogeneity

30 in the form of discrete zones with homogeneous properties. Based on an assessment of dome rock heterogeneity,
31 we explore whether material property heterogeneity ("diffuse" heterogeneity) within these discrete zones can
32 promote dome instability. X-ray computed tomography shows that dome samples are characterised by high
33 microstructural heterogeneities; e.g., porosity varies from 0.07 to 0.20 over millimetric lengthscales. To explore
34 how microstructural heterogeneity influences sample-scale strength, we performed numerical simulations using
35 Rock Failure and Process Analysis. The mean mechanical properties of the numerical samples were constant, and
36 we introduced heterogeneity by varying their distribution using a Weibull probability function. The models show
37 that increasing heterogeneity can reduce sample-scale strength by more than a factor of two. To explore the
38 influence of dome-scale heterogeneity, we numerically generated lava domes in Particle Flow Code. The domes
39 have the same bulk strength but are characterised by different degrees of heterogeneity by varying the distribution
40 of cohesion using a Weibull probability function. The models show that a greater degree of heterogeneity induces
41 higher dome-scale displacements and that, when there is also a discrete weakened zone, the addition of diffuse
42 heterogeneity leads to more widely distributed deformation. Therefore, alongside discrete zones defined by
43 different material properties, we find that the diffuse heterogeneity inherent to a dome is sufficient to compromise
44 dome stability and should be incorporated in future modelling endeavours.

45

46 **Keywords:** lava dome; heterogeneity; uniaxial compressive strength; porosity; coefficient of variation

47

48 **1 Introduction**

49 The collapse of a lava dome, which can result in rock falls, debris avalanches, laterally-directed
50 explosions, and pyroclastic density currents, represents a significant volcanic hazard (Sparks, 1997; Nakada et al.,
51 1999; Voight, 2000; Voight et al., 2000; Voight and Elsworth, 2000; Calder et al., 2002; Saucedo et al., 2005;
52 Komorowski et al., 2013; Ogburn et al., 2015; Harnett et al., 2019a). The collapse of a lava dome can also rapidly
53 depressurise subsurface magma, triggering large and potentially devastating eruptions (Sparks et al., 2002). The
54 mechanisms of dome instability are numerous, and very often operate in concert. For example, dome stability can
55 be compromised by increases in magma extrusion rate (Calder et al., 2002), basal strike-slip movements (Lagmay
56 et al., 2000), the build-up of pressurised fluids (Sparks, 1997; Komorowski et al., 1997; Voight and Elsworth,
57 2000; Reid, 2004; Ball et al., 2018; Heap et al., 2021a; Mordensky et al., 2022), topographical, geometrical, and
58 structural factors (Walter et al., 2015; Darmawan et al., 2018; Harnett and Heap, 2021; Mériaux et al., 2022),
59 ground acceleration resulting from earthquakes (Wallace et al., 2021), hydrothermal alteration (Reid et al., 2001;

60 Ball et al., 2013; Heap et al., 2021b; Darmawan et al., 2022; Harnett et al., 2022), and rain-induced pore-fluid
61 pressurisation (Matthews et al., 2002; Elsworth et al., 2004; Taron et al., 2007).

62 A characteristic common to all lava domes is that they are heterogeneous across a variety of scales. At
63 the microstructural scale (tens to hundreds of microns), microscopy and X-ray computed tomography have shown
64 that volcanic rocks, including dome rocks, are heterogeneous, typically containing a groundmass populated by
65 phenocrysts and pores in variable quantities and of variable sizes and shapes (Zandomeneghi et al., 2010; Shea et
66 al., 2010; Kushnir et al., 2016; Farquharson et al., 2016; Rhodes et al., 2018; Cashman, 2020). Moreover, the
67 observation of micro-breccia and micrometre-size pyroclast-filled fractures (tuffisites) in cryptodomes and lavas
68 (Heiken et al., 1988; Komorowski et al., 1997; Tuffen et al., 2003) are evidence of the presence of textural
69 discontinuities and the circulation of pressurised gas that may generate variations in mechanical properties and,
70 therefore, heterogeneity. At the sample scale (a few to tens of centimetres), laboratory studies have shown that the
71 physical and mechanical properties of rocks collected from the same dome can vary by several orders of magnitude
72 (Smith et al., 2011; Heap et al., 2014a; Farquharson et al., 2015; Harnett et al., 2019b; Bain et al., 2019; Heap et
73 al., 2020a; Kendrick et al., 2021; Heap and Violay, 2021). The large range in the physical and mechanical
74 properties typically observed for volcanic rocks of the same porosity is often considered to be the result of micro-
75 and sample-scale heterogeneities (e.g., Heap and Violay, 2021). Numerical experiments using modelling software
76 Fast Lagrangian Analysis of Continua (FLAC; Itasca Consulting Group Ltd) (Villeneuve et al., 2012), UDEC and
77 3DEC (Distinct Element Modelling; Itasca Consulting Group Ltd) (Ghazvinian et al., 2014; Stavrou et al., 2019;
78 Xu et al., 2020), Rock Failure Process Analysis (RFPA; Mechsoft Technology) (Tang et al., 2000, 2007; Zhu,
79 2008), and Particle Flow Code (PFC; Itasca Consulting Group Ltd) (Peng et al., 2017; Liakas et al., 2017) have
80 also demonstrated that microstructural heterogeneity leads to a reduction in rock strength. Numerical experiments
81 have also shown that pore size and shape (Heap et al., 2014b; Griffiths et al., 2017; Heap et al., 2021c) and crystal
82 content (Heap et al., 2016) can influence mechanical behaviour of volcanic rocks.

83 At the dome-scale (tens to hundreds of metres), field investigations and analogue and numerical
84 modelling have shown that lava domes are structurally very complex, with mechanical properties that vary through
85 space and time (Fink and Griffiths, 1998; Watts et al., 2002; Harnett et al., 2019b; Zorn et al., 2020; Tsepelev et
86 al., 2021). Geophysical methods have also exposed the large-scale heterogeneity of lava domes. For example,
87 muon tomography—a geophysical method that can provide the 3D density structure of a volcano—has shown that
88 the density of a dome can be spatially very variable (Tanaka et al., 2007; Lesparre et al., 2012; Rosas-Carbajal et
89 al., 2017), corresponding to spatial variations in compressive and tensile strength (Heap et al., 2021b). Electrical

90 resistivity and magnetic surveys have shown that portions of a lava dome can be affected by the presence of a
91 hydrothermal system, the location and/or extent of which can depend on internal structural features (Rosas-
92 Carbajal et al., 2016; Byrdina et al., 2017; Finn et al., 2018; Peterson et al., 2021). Heterogeneity is commonly
93 reflected in numerical and analogue models of volcanoes and volcanic domes, typically designed to better
94 understand large-scale deformation (e.g., slow spreading, sliding on pre-existing faults or fractures, the formation
95 of large-scale fractures, partial collapse), as discrete zones defined by different material properties. For example,
96 analogue models have used different materials—silicon putty to represent weakened or altered zones and a sand-
97 gypsum mixture to represent stronger and/or intact zones—to represent different discrete zones within the volcano
98 (Cecchi et al., 2004; van Wyk de Vries et al., 2000, Byrne et al., 2013). Numerical models typically separate the
99 volcanic edifice, flank, or dome into different discrete zones, often guided by geophysical data, and assign these
100 zones different rock physical and mechanical properties (Heap et al., 2021b; Harnett and Heap, 2021; Carr et al.,
101 2022; Harnett et al., 2022; Mordensky et al., 2022). An example of this *discrete heterogeneity* is shown in Fig. 1a,
102 where a lava dome is split into two discrete zones characterised by different, but uniform, physical and mechanical
103 properties (a weak zone in yellow and a strong zone in white). In the context of a volcanic edifice, flank, or dome,
104 such discrete zones could represent, for example, volumes that are characterised by different rock types with
105 different properties, different lava units, different domes or spines within a dome complex, or zones variably
106 influenced by hydrothermal alteration. However, existing geophysical datasets suggest that, although volcanic
107 structures can, and should, be considered with discrete zones characterised by different physical and mechanical
108 properties, these properties also vary within these discrete zones (termed here as *diffuse heterogeneity*). An
109 example of *discrete and diffuse heterogeneity* is shown in Fig. 1b, where the lava dome is again split into two
110 discrete zones characterised by different physical and mechanical properties (i.e. *discrete heterogeneity*) but, in
111 this instance, the physical and mechanical properties within these zones are also heterogenous (i.e. *diffuse*
112 *heterogeneity*). However, the extent to which diffuse heterogeneity can influence dome stability remains
113 unexplored.

114 Our goal in this contribution is to better understand the role of heterogeneity on the stability of a lava
115 dome. We first characterised the microstructural heterogeneity of dome rocks using X-ray micro computed
116 tomography (μ CT). We then performed numerical experiments, calibrated using laboratory stress-strain data for
117 dome rock, to assess the influence of systematically varied microstructural heterogeneity on the strength of dome
118 rocks at the sample-scale. Finally, we investigated the effect of heterogeneity at the dome scale. To do so, we
119 simulated domes characterised by the same average bulk strength, but with different heterogeneities (i.e. diffuse

120 heterogeneity), and domes characterised by both discrete and diffuse heterogeneities. These results reveal new
121 insights on the influence of the internal structure and fabric of volcanic domes on their stability. As such they are
122 aimed at providing one of the missing links to progress beyond the state of the art of modelling and the quantitative
123 evaluation of dome volumes susceptible to instability and collapse and the modelling of resulting rockslide and
124 debris avalanche emplacement as proposed by, for example, Peruzzetto et al. (2019). Understanding the
125 mechanisms that can contribute to dome instability and collapse will help improve monitoring strategies and
126 mitigate the hazards presented by lava domes worldwide.

127

128 **2 Materials and Methods**

129 2.1 Microstructural and compositional heterogeneity

130 To assess the microstructural heterogeneity of dome rocks, X-ray tomography images were acquired for
131 cylindrical samples (10 mm in diameter and 20 mm in length) prepared from three dome rocks from La Soufrière
132 de Guadeloupe, an active andesitic stratovolcano located on the French island of Guadeloupe in the Eastern
133 Caribbean (Moretti et al., 2020). Two of the three dome rocks were collected from a collapse scar to the northeast
134 of dome summit (samples H3 and H4A), and the third was collected from the dome summit near Cratère Sud
135 (sample H19). Samples from these blocks were recently analysed in terms of their compressive and tensile strength
136 (Heap et al., 2021b, 2022a, b) and thermal properties (Heap et al., 2022c). Fig. 2 shows backscattered scanning
137 electron microscope (SEM) images of each sample. All three dome rocks are porphyritic andesites characterised
138 by a microcrystalline groundmass with phenocrysts of dominantly plagioclase and pyroxene (orthopyroxene and
139 clinopyroxene) (Fig. 2). The connected porosity of each of the three cylindrical samples was calculated using their
140 bulk sample volumes and their skeletal (connected) volumes measured using a helium pycnometer. The connected
141 porosities were measured to be 0.18, 0.20, and 0.12 for samples H3, H4A, and H19, respectively.

142 X-ray tomography images were acquired for the three cylindrical samples at the High-Resolution X-ray
143 Computed Tomography Facility at the University of Texas at Austin (USA). X-rays were set to 90 kV and 12 W,
144 and the acquisition time was 0.07 s. A total of 4175 image slices, with a voxel side length of 5 μm , were taken of
145 each of the three cylindrical samples.

146 X-ray powder diffraction (XRPD) was used to provide the mineral content of all three samples (data from
147 Heap et al., 2021b, 2022c). These XRPD data reveal that all three samples are characterised by advanced argillic
148 alteration, the result of the efficient circulation of acidic sulfate-chloride-rich fluids (below 350 °C, and down to
149 150–200 °C; pH < 4) (Heap et al., 2021b). Sample H3 contains primary plagioclase (46.6 wt%), pyroxene (11.8

150 and 5.6 wt% of orthopyroxene and clinopyroxene, respectively), and magnetite (0.8 wt%); secondary minerals
151 include kaolinite (17.4 wt%), cristobalite (10.6 wt%), pyrite (3.8 wt%), Na-alunite (2.8 wt%), and quartz (0.6
152 wt%). Sample H4A contains primary plagioclase (23.3 wt%) and pyroxene (11.8 and 4.9 wt% of orthopyroxene
153 and clinopyroxene, respectively); secondary minerals include kaolinite (43.4 wt%), cristobalite (11.8 wt%), pyrite
154 (2.3 wt%), Na-alunite (1.3 wt%), gypsum (0.7 wt%), and quartz (0.6 wt%). Sample H19 contains primary
155 plagioclase (22.0 wt%) and pyroxene (10.2 and 5.0 wt% of orthopyroxene and clinopyroxene, respectively),
156 secondary minerals include opal-A (33 wt%), Na-alunite (14.2 wt%), cristobalite (9.5 wt%), hematite (2.4 wt%),
157 kaolinite (2.0 wt%), and quartz (1.7 wt%). In all three samples, Na-alunite and cristobalite are observed as pore-
158 filling precipitates and plagioclase phenocrysts are often altered and partially replaced by opal-A and/or kaolinite
159 (Fig. 2). The total wt% of secondary (alteration) minerals, a measure of their alteration, in samples H3, H4A, and
160 H19 is 35.2, 60, and 62.8 wt%, respectively. We highlight that, although these hydrothermally altered samples are
161 representative of the material forming the dome at La Soufrière de Guadeloupe, they may not represent freshly
162 extruded dome materials at other active volcanoes that can be characterised by initially low levels of alteration.

163

164 2.2 Sample scale heterogeneity: Rock Failure and Process Analysis modelling

165 To explore the effect of heterogeneity on the sample scale, 2D numerical experiments were performed
166 using Rock Failure and Process Analysis (RFPA), a numerical code based on elastic damage mechanics (Tang,
167 1997, 2011). Numerical samples were prepared to the same dimensions as the uniaxial compressive strength tests
168 performed in Heap et al. (2021b), which were 20 mm in diameter and 40 mm in length. Before we performed the
169 numerical experiments, we ran some preliminary simulations to ascertain the appropriate mesh size for the model.
170 To do so, we performed simulations with the aim of using the model to capture stress-strain data from a laboratory
171 experiment performed on a sample of andesite from La Soufrière de Guadeloupe (sample H18; Heap et al., 2021b)
172 in which the numerical samples consisted of either 45,000 or 80,000 square elements, resulting in resolutions of
173 7.5 and 10 elements per mm, respectively. Both of these simulations accurately captured the mechanical behaviour
174 of the laboratory experiment and so we used numerical samples containing 45,000 square elements (resolution of
175 7.5 elements per mm) for our numerical experiments. In the model, each square element is assigned a set of
176 physical and mechanical input parameters. To obtain the mean values for these model input parameters, we
177 performed a calibration in which we modified the model input parameters until the stress-strain behaviour,
178 Young's modulus, and uniaxial compressive strength from the numerical experiment matched that of a laboratory
179 experiment performed on a sample of andesite from La Soufrière de Guadeloupe (sample H18; Heap et al., 2021b).

180 The stress-strain curves for the numerical and laboratory experiment are shown in Fig. 3a, and the mean input
181 parameters for the calibrated model are listed in Table 1. For the calibration model, the physical and mechanical
182 properties of each square element were not identical, and varied according to a Weibull probability density
183 function:

184

$$185 \quad x(u) = \frac{m}{u_0} \left(\frac{u}{u_0} \right)^{m-1} \exp \left[- \left(\frac{u}{u_0} \right)^m \right]. \quad (1)$$

186

187 A Weibull probability density function contains a scale parameter and a shape parameter. Here, u is the scale
188 parameter of an individual element, and the scale parameter u_0 is the average element parameter. $x(u)$ is the
189 dependence of a given property on the distribution scale input, u , such that $x(u)$ is replaced by the Young's
190 modulus, $E_0(u)$, or the uniaxial compressive strength, $\sigma_c(u)$, to determine the probability of a certain square
191 element having a certain property. The shape parameter is m . In material sciences, the shape parameter m is often
192 called the "*homogeneity index*". For the calibration model, we set m to be 5. As a result, although the mean uniaxial
193 compressive strength of an element in the numerical experiment shown in Fig. 3a was 147.3 MPa (Table 1), the
194 strength of an individual element varied according to Eq. (1), where $m = 5$. This microscale heterogeneity resulted
195 in a numerical sample with a macroscopic uniaxial compressive strength of 98.8 MPa (Fig. 3a). Therefore,
196 changing homogeneity index, m , and keeping all the other parameters in the model constant, allows us to explore
197 the influence of microscale heterogeneity on sample-scale uniaxial compressive strength, where high values of m
198 correspond to very homogeneous samples, and low values of m correspond to very heterogeneous samples. An
199 extremely high homogeneity index yields a Gaussian distribution with high kurtosis (Xu et al., 2018). As the
200 homogeneity index decreases, the distribution becomes increasingly positively skewed (Fig. 3b). Indeed, although
201 the mean values remain constant, the median values of uniaxial compressive strength and Young's modulus
202 decrease as a function of decreasing homogeneity index. For example, at $m = 1.1$, the median uniaxial compressive
203 strength and Young's modulus are 105.7 MPa and 33.2 GPa, respectively. Owing to their success in simulating
204 the mechanical behaviour and failure of rock, the statistics of the mechanical properties of rock are often assumed
205 to follow Weibull distributions in numerical modelling (Tang, 1997; Zhu and Tang, 2004; Liu et al., 2004; Wong
206 et al., 2006; Tang, 2011). Weibull distributions have also been used in other aspects of volcanology. For example,
207 Weibull functions have been used to describe the distribution of dyke thickness (Krumbholz et al., 2014) and the
208 distribution of flaws surrounding magma chambers (Gray and Monaghan, 2004).

209 We performed numerical experiments in which deformed samples were characterised by different values
210 of m : 1.1 (most heterogeneous), 2, 3, 4, 5, 6, 7, and 8 (most homogeneous). The distribution of Young's modulus
211 (mean value of 46 GPa; Table 1) and uniaxial compressive strength (mean value of 147.3 MPa; Table 1) for each
212 of these numerical experiments are shown in Figs. 3b and 3c, respectively. For example, as shown in Fig. 3c, the
213 elements comprising the sample with the highest value of m are clustered around the mean value of 147.3 MPa,
214 whereas the elements comprising the sample with the lowest value of m can be much lower (close to 0 MPa) or
215 much higher (higher than 350 MPa) than this mean value. We note that the microstructural heterogeneity in our
216 numerical samples is provided by varying the physical and mechanical properties of square elements that have a
217 length of 133 μm and, as a result, the modelling assumes microstructural heterogeneities of the same size. In nature,
218 and in our dome samples (Fig. 2), volcanic rocks can contain heterogeneities on a number of scales, from the nano-
219 or microscale to the scale of the sample (centimetric scale). The influence of larger heterogeneities, such as mm-
220 scale pores (Heap et al., 2014b; Griffiths et al., 2017; Heap et al., 2021c) and crystals (Heap et al., 2016), on the
221 mechanical behaviour of volcanic rocks have been investigated in previous studies using a heterogeneous
222 groundmass with a constant homogeneity index ($m = 3$). The modelling presented here varies the homogeneity
223 index of the groundmass in numerical samples ($m = 1.1, 2, 3, 4, 5, 6, 7, \text{ and } 8$) that do not contain larger pores and
224 crystals, or any other mm-scale heterogeneity.

225

226 2.3 Dome scale heterogeneity: Particle Flow Code modelling

227 To explore the effect of heterogeneity on the dome scale, we used 2D Discrete Element Method (DEM)
228 models created in Particle Flow Code (PFC; Itasca Consulting Group Ltd), following the methods outlined by
229 Harnett et al. (2018) and Harnett and Heap (2021). The DEM allows us to model the post-failure detachment and
230 movement of blocks of rock, which cannot be simulated by the FEM-based software RFPa used for the sample-
231 scale modelling. We suggest that DEM modelling is therefore more relevant for dome-scale simulations due to the
232 spectrum of collapse processes that can be expected at natural domes (e.g., Calder et al., 2002). Indeed, PFC
233 modelling has been previously, and successfully, used to model dome growth and collapse (Husain et al., 2014;
234 Harnett et al., 2018; Harnett and Heap, 2021; Harnett et al., 2022). These DEM models consider a particle-based
235 material in which circular particles interact at interparticle contacts. Contact behaviour is primarily governed by
236 stiffness and cohesion. At the boundary between the particles and walls (i.e. the ground surface), the contact
237 behaviour is cohesionless and governed by friction. The particle size in the model is not representative of individual
238 crystals, grains, or rock blocks, but rather represents discretisation of the medium for the purpose of computation.

239 First, we numerically generated two fully solid domes (see Harnett and Heap, 2021) containing 9745
240 circular particles and 22710 contacts. These contacts are given a uniform strength of 1 MPa. We select here a much
241 lower bulk strength for the dome-scale (1 MPa) than the sample-scale (tens to hundreds of MPa) following previous
242 studies that have shown that a decrease in magnitude is required when upscaling laboratory parameters for use at
243 the dome-scale (Walter et al., 2019; Villeneuve and Heap, 2021). The domes are stable, such that they will not
244 deform unless perturbed in some way.

245 To explore the effect of heterogeneity on the stability of a dome, we used the Weibull probability density
246 function shown in Eq. 1 to provide a distribution of the cohesion parameter acting at each interparticle contact.
247 For one dome, we let $m = 2$, creating a heterogenous dome, where the lowest contact cohesion is close to 0 MPa
248 (representative of weak or altered rock) and the highest contact cohesion is 3 MPa (representative of strong or
249 fresh rock). For the other dome, we let $m = 10$, creating a relatively homogeneous dome, where the contact
250 cohesion values cluster more tightly around 1 MPa, with minimum cohesion values of ~ 0.5 MPa and maximum
251 cohesion values of < 1.5 MPa. In both cases, the average strength remains equal (i.e., the mean bond strength is 1
252 MPa in both cases), allowing us to solely explore the influence of heterogeneity on dome strength. In each case,
253 we vary both the cohesive and tensile strength as a function of the Weibull distribution, where the compressive to
254 tensile strength ratio is constant at 10:1 (Heap et al., 2022a). The distributions of bond cohesion for the two domes
255 are shown in Fig. 4.

256

257 **3 Results**

258 3.1 Microstructural and compositional heterogeneity

259 We used X-ray micro computed tomography (μ CT) to assess the microstructural and compositional
260 heterogeneity of three dome samples from La Soufrière de Guadeloupe. Using open-source software ImageJ, we
261 first plot grey level histograms for each sample using the entire μ CT image volumes (Fig. 5). The grey level is a
262 value between 0 (lowest attenuation and lowest density) and 255 (highest attenuation and highest density). The
263 three dome rocks are characterised by three peaks: at values of grey level of ~ 20 , ~ 45 – 50 , and ~ 90 – 95 (Fig. 5).
264 Although we consider the peak at a grey level of ~ 20 contains to correspond to the porosity within the sample, this
265 peak also includes voxels that contain both porosity and solid material (groundmass and/or crystals). The peaks at
266 grey levels of ~ 45 – 50 and ~ 90 – 95 likely correspond to, respectively, the groundmass and higher-density crystals
267 (e.g., plagioclase crystals that are denser than the groundmass). The broad nature of these peaks, and the saddles

268 between them, are likely the result of voxels containing mixtures of groundmass and crystals. The wide variation
269 in grey level, and therefore density, within the samples speaks to their high microstructural heterogeneity.

270 Next, we stacked the μ CT images in ImageJ and determined the coefficient of variation (COV) of the
271 grey level (the ratio of the standard deviation of the grey level to the mean grey level), the porosity, the average
272 equivalent pore diameter, the average pore aspect ratio, and the average pore circularity for each image in the stack
273 (a total of 4175 image slices). The COV of the grey level in μ CT images of rocks has previously been used to
274 assess microstructural heterogeneity (e.g., Louis et al., 2007). In these analyses, and due to edge effects, we
275 removed 300 slices (i.e. 1.5 mm) from the top and bottom of each of the stacks. The equivalent pore diameter, d ,
276 was determined using $d = (3/2)d_F$, where d_F is the average Feret diameter. The pore aspect ratio is the ratio of
277 the minor to major pore axis, where a perfect circle would have an aspect ratio of one. Finally, the pore circularity,
278 C , is given by $C = 4\pi p^2$, where p is the pore perimeter. A perfect circle will have a pore circularity of one. In all
279 the analyses of pore size and shape, we excluded all pores that were determined to have a diameter $< 10 \mu\text{m}$.

280 Using the voxel side length of $5 \mu\text{m}$ as the thickness of each image, we can plot these microstructural
281 parameters as a function of sample length. We plot the COV and porosity as a function of sample length for each
282 of the three dome samples in Figs. 6 and 7, respectively (plots for the average equivalent pore diameter, the average
283 pore aspect ratio, and the average pore circularity are provided in the Supplementary Information). We also provide
284 the image slice with the highest and lowest value for each sample in Figs. 6 and 7. Fig. 6 shows that the COV
285 varies from 0.27–0.31 to 0.48–0.52, and that the variation in COV is approximately the same for each of the three
286 samples. The image slices with the lowest values of COV are those without high-density pyrite and hematite and
287 with few plagioclase phenocrysts (Fig. 6). Conversely, the image slices with the highest values of COV are those
288 with large amounts of high-density pyrite and hematite and large plagioclase phenocrysts (Fig. 6). Fig. 7 shows
289 that the porosity of samples H3, H4A, and H19 varies from 0.07–0.20, 0.04–0.10, and 0.05–0.11, respectively.
290 The image slices with the lowest and highest porosities are those with a small and large number of macropores,
291 respectively (Fig. 7). The average porosity calculated from the stacked CT images are 0.11, 0.07, and 0.08 for H3,
292 H4A, and H19, respectively (Fig. 7). These calculated porosities are lower, and much lower in the case of H4A,
293 than the connected porosities measured in the laboratory (0.18, 0.20, and 0.12 for H3, H4A, and H19, respectively).
294 The lower porosities calculated using the CT images, compared to the laboratory-measured values, is the result of
295 the presence of $< 10 \mu\text{m}$ -diameter micropores that are, presumably, abundant in sample H4A.

296 We find that the average equivalent pore diameters of samples H3, H4A, and H19 are 82, 63, and 99 μm ,
297 respectively. The average pore aspect ratio of each sample is 0.52. And, finally, the average pore circularities of

298 samples H3, H4A, and H19 are 0.73, 0.74, and 0.68, respectively. We find that the equivalent pore diameter, the
299 pore aspect ratio, and the pore circularity do not vary considerably along the sample lengths (see data in
300 Supplementary Information). Although the proportion and size of the macropores varies from slice to slice (see
301 the image slices in Fig. 7), the large number of smaller pores results in a similar average equivalent pore diameter
302 for each of the slices along the length of the samples.

303

304 3.2 Impact of heterogeneity on the sample scale

305 To explore the effect of microstructural and compositional heterogeneity on the sample scale, we
306 performed calibrated 2D numerical experiments using RFPA. The samples in these numerical experiments were
307 all characterised by the same average Young's modulus and strength, but we varied their homogeneity by changing
308 the range of these two parameters using a Weibull probability density function (Fig. 2; Eq. 1). The results of the
309 numerical experiments are shown in Fig. 8 (see also Table 2). We first note that all the numerical samples failed
310 in a brittle manner, whereby sample failure was promoted by the formation of a throughgoing macroscopic fracture
311 (see inset on Fig. 8b). The data of Fig. 8 show that the uniaxial compressive strength of the numerical samples
312 decreases from ~ 110 MPa at a homogeneity index of 8 (the most homogeneous sample) to ~ 45 MPa at a
313 homogeneity index of 1.1 (the most heterogeneous sample). In other words, increasing sample heterogeneity can
314 reduce sample-scale strength by more than a factor of two. Fig. 8 also shows that the reduction in strength as the
315 homogeneity index is decreased is non-linear: strength is decreased by only a couple of MPa when the homogeneity
316 index is decreased from 8 to 7, but decreases by almost 20 MPa when the homogeneity index is decreased from 2
317 to 1.1 (see also Table 2). We also highlight that the post-peak portion of the numerical experiments performed at
318 $m = 7$ and 8 are different to those performed at lower values of m (Fig. 8). The stress drop associated with
319 macroscopic sample failure at $m < 7$ occurs almost immediately after the peak stress is reached whereas, at $m = 7$
320 and 8, the sample reaches the peak and there is an appreciable strain softening period prior to the stress drop that
321 accompanies macroscopic failure (Fig. 8).

322

323 3.3 Impact of heterogeneity on the dome-scale

324 To explore the effect of heterogeneity at the dome scale, we performed 2D simulations using PFC. The
325 model starting condition in each stage is a quasi-stable dome, which has already settled under gravitational forces
326 and will not deform unless intrinsic or extrinsic factors are changed. The initial modelled dome has a width of 440
327 m and a maximum height of 130 m. The new models presented here show domes characterised by the same average

328 bond cohesion; this is analogous to the uniaxial compressive strength of the macroscale material. To explore the
329 influence of heterogeneity, we prepared a reasonably homogenous dome (for which $m = 10$; Fig. 4; Eq. 1) and
330 a very heterogeneous dome (for which $m = 2$; Fig. 4; Eq. 1). The distribution of bond cohesion in these models
331 is shown in Fig. 9. The domes analysed in this study are unbuttressed domes that were emplaced on a flat surface
332 (Fig. 9).

333 The influence of dome heterogeneity on an otherwise-stable dome is presented in Fig. 10. The images in
334 Fig. 10 show the displacement in the dome when the contact cohesion between particles was changed from a
335 uniform value of 1 MPa (the quasi-stable dome; perfectly homogeneous in terms of particle contact cohesion) to
336 values assigned by the Weibull probability function (mean bond strength in both scenarios is 1 MPa; but contact
337 cohesion varies from ~ 0.5 MPa to <1.5 MPa for the relatively homogenous dome (Fig. 10a) and from close to 0
338 MPa to 3 MPa for the heterogeneous dome (Fig. 10b)). The average displacement is 0.0036 m for the relatively
339 homogenous dome (Fig. 10a) and 0.0198 m for the heterogeneous dome (Fig. 10b). The dome with higher
340 heterogeneity shows higher displacements across the dome, and particularly at its centre (Fig. 10b), and shows that
341 heterogeneity alone is enough to destabilise the dome, even when the bulk strength remains equal. By contrast, the
342 relatively homogeneous dome remains stable, with only minor displacements toward the left flank of the dome
343 (Fig. 10a).

344

345 **4 Discussion**

346 4.1 Microstructural heterogeneity of volcanic rocks

347 Our SEM images (Fig. 2) and μ CT data (Figs. 5, 6, and 7) have shown that dome rocks can be
348 characterised by a high degree of microstructural heterogeneity. As a comparison, the COV values calculated for
349 the four sandstones analysed in Louis et al. (2007), using images acquired at a very similar resolution to those
350 presented here (a voxel side length of $5.7 \mu\text{m}$, compared to $5 \mu\text{m}$ for the images acquired for this study), were
351 0.186, 0.255, 0.303, and 0.313, respectively. The maximum COV values determined for our three dome rocks
352 were 0.48, 0.52, and 0.51 for H3, H4A, and H19, respectively (Fig. 6). The domes rock from La Soufrière de
353 Guadeloupe were also characterised by three broad X-ray attenuation coefficient peaks associated with
354 macropores, plagioclase phenocrysts, and high-density Fe-Ti oxides (Fig. 5), as also observed for dome rock from
355 Volcán de Colima (Mexico; Heap et al., 2020b). By contrast, X-ray attenuation coefficient peaks in porous
356 sandstones are typically strongly bimodal, with narrow peaks associated with macropores and grains (Louis et al.,
357 2007). Two X-ray attenuation coefficient peaks are also observed in porous limestones (Ji et al., 2015). We note

358 that the range of COV values for the more altered dome rocks (H4A and H19; characterised by an alteration of 60
359 and 62.8 wt.%, respectively) is larger than for the least altered dome rock (H3; characterised by an alteration of
360 35.2 wt.%) (Fig. 6), suggesting that hydrothermal alteration may increase the heterogeneity of dome rock (although
361 more data are now required to test this hypothesis).

362 The variation in porosity along the length of a 10 mm-long sample of dome rock is particularly noteworthy
363 (Fig. 7). We find that the porosity of samples H3, H4A, and H19 varies from 0.07–0.20, 0.04–0.10, and 0.05–0.11,
364 respectively (Fig. 7). For comparison, the porosity measured using μ CT data along the length of sandstone samples
365 was found to be $\sim 0.04 \pm 0.002$ (Long et al., 2009), $\sim 0.3 \pm 0.03$ (Appoloni et al., 2007), and the porosity of a suite
366 of low-porosity (< 0.1) sandstones was found to vary by up to about ± 0.025 (Zhao et al., 2019). The porosities of
367 three porous limestones were found to be 0.077–0.103 (average 0.1), 0.170–0.236 (average 0.2), and 0.237–0.288
368 (average 0.26) (Baud et al., 2017). Because porosity plays a first-order role in dictating physical properties such
369 as sample strength and Young's modulus, the wide range of porosity found within a 10 mm-long sample questions
370 the use of the average sample porosity when interpreting these data, and could help explain the large scatter in
371 strength and Young's modulus when plotted as a function of sample porosity (Heap and Violay, 2021). In other
372 words, samples with the same average sample porosity likely have different strengths simply due to whether the
373 porosity is homogeneously or heterogeneously distributed throughout the sample. Finally, we note that the
374 difference in porosity between that calculated from the stacked CT images and that measured in the laboratory
375 (Fig. 7) implies that the dome samples studied herein contain significant microporosity (pores that are $< 10 \mu\text{m}$ in
376 diameter). This is particularly true for sample H4A (Fig. 7).

377 To conclude, and as discussed in Heap and Violay (2021), volcanic rocks are typically much more
378 microstructurally heterogeneous than, for example, granites or sedimentary rocks. For example, the porosity of
379 volcanic rocks can vary significantly (from almost 0 to almost 1), they can contain a wide distribution of pore sizes
380 that can be characterised by different geometries (Shea et al., 2010; Zandomeneghi et al., 2010), they can contain
381 both pores and microcracks (Kushnir et al., 2016), they can contain a variety of different types of crystals in
382 different abundances that can be characterised by different crystal sizes, shapes, and size distributions (Cashman,
383 2020), their groundmass can be variably crystallised (Clarke et al., 2007; Zorn et al., 2018), and they can be
384 variably influenced by hydrothermal alteration (Heap et al., 2021b).

385

386 4.2 Impact of heterogeneity on the strength of volcanic rocks and dome stability

387 Our calibrated 2D numerical experiments have shown that heterogeneity alone can reduce the uniaxial
388 compressive strength of dome rock by more than a factor of two (Fig. 8). Previous modelling studies also found
389 that microstructural heterogeneity leads to a reduction in rock strength (Tang et al., 2007, 2000; Zhu, 2008;
390 Villeneuve et al., 2012; Ghazvinian et al., 2014; Peng et al., 2017; Liakas et al., 2017; Xu et al., 2020), and
391 modelling designed to investigate the mechanical behaviour of volcanic rocks showed that pore size and shape
392 (Heap et al., 2014b; Griffiths et al., 2017) and crystal content (Heap et al., 2016) can also influence rock strength.
393 In the RFPA model, the numerical samples characterised by a high heterogeneity contain weaker elements (Fig.
394 3), which fail at lower stresses. The failure of a weak element results in a stress redistribution that can damage or
395 fail neighbouring elements, even those with higher strengths. As a result, heterogeneity can provide weak elements,
396 or weak zones, that can act as the locus for the formation of larger fractures that can result in macroscopic sample
397 failure. In terms of a volcanic rock, microstructural heterogeneities such as pores and fractured crystals or, in the
398 case of altered volcanic rocks, crystals that have suffered dissolution or partial or complete replacement with weak
399 minerals such as clays, could act as the loci for damage that eventually results in macroscopic sample failure.

400 Our 2D dome simulations have shown that diffuse heterogeneity alone can promote lava dome instability
401 (Fig. 10). Lava domes are structurally complex and comprise rocks with different strengths and, as such, are
402 inherently heterogeneous. Our dome simulations (Fig. 10) highlight the need for not only assessing dome
403 heterogeneity, but also monitoring processes that act to increase the heterogeneity of a dome, including processes
404 that increase heterogeneity without changing the bulk strength of the dome. For example, the circulation of
405 hydrothermal fluids may act to remove precipitates and/or sublimates within fractures in one part of the dome,
406 resulting in weakening, which are then precipitated within fractures elsewhere in the dome, resulting in
407 strengthening (although this will depend on the nature of the precipitated minerals). Although the bulk strength of
408 the dome may not be greatly affected in this scenario, the increase in heterogeneity can weaken the entire structure,
409 resulting in instability and potentially collapse.

410 Dome-scale displacement as a result of increased diffuse heterogeneity shows downward and outward
411 movement of previously stable dome material (Fig. 10). This type of displacement was also found when modelling
412 the stability of a dome containing a discrete zone characterised by a lower, but uniform, strength (Harnett et al.,
413 2022), although displacement was focussed toward the flank of the dome where the weakened zone was located.
414 We suggest, therefore, that small magnitude, but widespread, deformation of a lava dome may be the result of
415 diffuse heterogeneity or may indicate a progressive increase in diffuse heterogeneity linked, perhaps, to a
416 progressive increase in hydrothermal alteration.

417 Based on the results of our study, we suggest that existing numerical models that simulate discrete zones
418 with different properties within a volcano need to also consider diffuse heterogeneity that might exist *within* these
419 zones. Material heterogeneity in analogue experiments could also be considered by using, for example, sand-sized
420 particles with different sizes and/or material properties. To demonstrate how discrete and diffuse heterogeneity
421 could influence the results of numerical models of dome stability, we provide the dome-scale displacement and
422 strain for (1) a homogenous dome with heterogeneous material properties ($m = 2$; Eq. 1) (Fig. 11a), (2) a dome
423 with an internal, off-centre weak zone in which both discrete zones have uniform material properties (Fig. 11b;
424 model results from Harnett et al., 2022), and (3) a dome with an internal, off-centre weak zone in which both
425 discrete zones have heterogeneous material properties ($m = 2$; Eq. 1) (Fig. 11c).

426 For the homogeneous dome with heterogeneous material properties, deformation and fracturing can be
427 observed on the flanks of the dome (Fig. 11a). Heterogeneous material properties induce an average displacement
428 of 0.18 m, while the maximum displacement for any one particle is 0.87 m. As discussed in Harnett et al. (2022),
429 a weak zone within the dome results in significant deformation and very large, dome-splitting fractures within and
430 above the off-centre weakened zone when both discrete zones have uniform material properties (Fig. 11b). In this
431 scenario, the introduction of a weakened discrete zone induces an average displacement of 0.29 m and a maximum
432 displacement of 3.22 m. When uniform material properties are considered in the modelling, deformation is very
433 much confined to the area adjacent to the weakened zone, and little to no deformation is observed on the opposite
434 flank (Fig. 11b). When the model from Harnett et al. (2022) (Fig. 11b) is re-run but, this time, incorporating
435 heterogeneous material properties in both zones ($m = 2$; Eq. 1), the deformation and fracturing within and above
436 the off-centre weakened zone is reduced, but there is more deformation and fracturing on the opposite flank of the
437 dome (Fig. 11c). The combined model showing both discrete and diffuse heterogeneity exhibits an average
438 displacement of 0.36 m and a maximum displacement of 1.42 m. Interestingly, the strain resulting from the diffuse
439 heterogeneity model (Fig. 11a) is not simply superimposed on the strain resulting from the discrete weakened zone
440 (Fig. 11b): when diffuse heterogeneity is considered, a weakened zone on one side of the dome results in more
441 fracturing on the opposite side of the dome (Fig. 11c). In other words, the introduction of diffuse heterogeneity in
442 a dome that also contains a discrete weakened zone has resulted in damage that is more distributed throughout the
443 dome, affecting a larger area, and less pronounced localisation in the form of dome-splitting fractures. This is
444 because, when diffuse heterogeneity is considered, there are weak, failure-prone contacts throughout the dome that
445 are not present when the material properties of the discrete zones are considered to be uniform. We speculate that,
446 although a reduction in dome-splitting fractures may in the short-term reduce the probability of partial flank

447 collapses, distributed dome deformation may, in the long-term, mobilise larger volumes and promote larger-scale
448 collapses.

449 These results highlight that deformation and fracturing within a dome may not necessarily indicate that
450 there is a proximal weak zone, potentially complicating monitoring strategies at active lava domes. Further
451 complications arise from the fact that this deformation pattern can be enhanced by the morpho-structural
452 relationship between the dome and its surroundings (i.e. a buttressing effect provided by the paleo-collapse
453 horseshoe-shape structure within which the dome was built or the inclination of the substratum of the dome; e.g.,
454 Heap and Harnett, 2021). Another important issue is the likely development of a collapse sequence with successive
455 retrogressive slide blocks retreating into the dome. Our new modelling (Fig. 11) also suggests that future dome
456 stability modelling should consider diffuse material property heterogeneity within discrete zones comprising the
457 modelled dome. An estimation of the different rock masses potentially involved in different collapse scenarios
458 (e.g., Peruzzetto et al., 2019) could also be refined to integrate the zonation of this dome heterogeneity. Ongoing
459 research is now underway to develop strategies and methods to improve the capacity and resolution of static and
460 functional imagery of volcanic domes (e.g., muon tomography and electrical conductivity), to find upscaling
461 relations between numerical models and the geophysical characterisation scale, and to couple the results with
462 thermo-hydro-mechanical simulations of fluid circulation within a lava dome.

463

464 **Concluding remarks**

465 The mechanisms considered responsible for dome instability and collapse are numerous and often operate
466 in concert. Although the mechanisms may vary, understanding their impact on stability typically involves
467 incorporating heterogeneity into the models, such as including material with a different rheology in analogue
468 models (van Wyk de Vries et al., 2000; Cecchi et al., 2004) or by lowering the strength of material within a specific
469 zone in numerical models (Harnett et al., 2022). The heterogeneities within these dome stability models are
470 modelled as discrete zones with different, but uniform, material properties. These discrete zones could represent,
471 for example, zones weakened by hydrothermal alteration, zones characterised by different rock types with
472 disparate physical and mechanical properties, or zones characterised by a higher fracture density or pressurised
473 fluids. Here, we consider the influence of diffuse heterogeneity on dome stability, motivated by the high
474 heterogeneity of volcanic rock samples. In other words, although zones of discrete heterogeneities exist, there is
475 also an underlying inherent heterogeneity in the material properties of a dome. Our numerical models show that
476 this inherent diffuse heterogeneity can result in dome instability, highlighting that (1) domes are inherently

477 unstable, (2) dome instability can occur in the absence of obvious discrete weakened and/or fractured zones hence
478 potentially producing volumes of unstable collapse-ready rock masses larger than otherwise expected, (3) when
479 diffuse heterogeneity is considered, the impact of an off-centre weakened zone within the dome is more distributed
480 throughout the dome (when compared to modelling that assumes uniform material properties for the discrete zones
481 comprising the dome), and (4) future numerical modelling should consider heterogeneity within the discrete zones
482 comprising the modelled dome. These results underscore the importance of understanding the range of volumes
483 of material that can be involved in collapse scenarios. Furthermore, an increase in the heterogeneity in lava domes
484 will likely reduce cohesion and, combined with other factors, may enhance the mobility of a collapse debris-
485 avalanche, thereby increasing its runout and hazard potential. Understanding the mechanisms that can contribute
486 to dome instability and collapse will help improve monitoring strategies and mitigate the hazards presented by
487 lava domes worldwide.

488

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501

502 **Statements and Declarations**

503 The authors declare no financial or non-financial interests that are directly or indirectly related to this work.

504

505 **Supplementary Information**

506 Our manuscript is accompanied by two Supplementary Information files. Supplementary Information file 1
507 contains additional figures showing equivalent pore diameter, pore aspect ratio, and pore circularity as a function
508 of sample length. Supplementary Information file 2 is an Excel spreadsheet containing the data for Figures 5, 6, 7,
509 and 8.

510

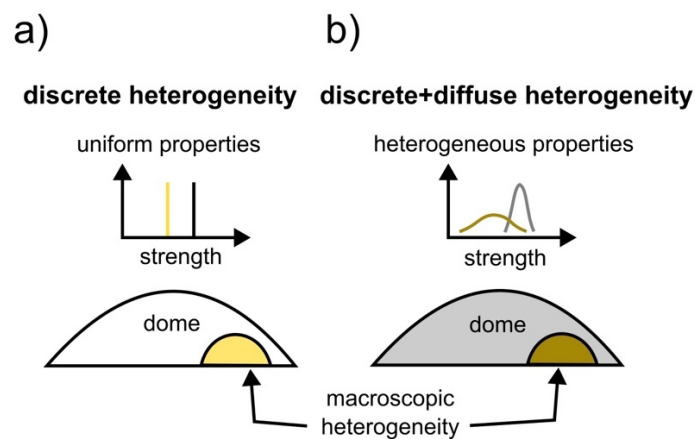
511 Author contributions

512 M. Heap and C. Harnett conceived and developed the idea for this study. Numerical experiments using PFC were
513 performed by C. Harnett. Numerical experiments using RFPA were performed by Z. Heng and T. Xu. Image
514 analysis was performed by T. Nazarbayov. M. Heap and C. Harnett wrote the first draft of the manuscript, with
515 contributions from all authors. All authors read and approved the final manuscript.

516

517 Figures

518

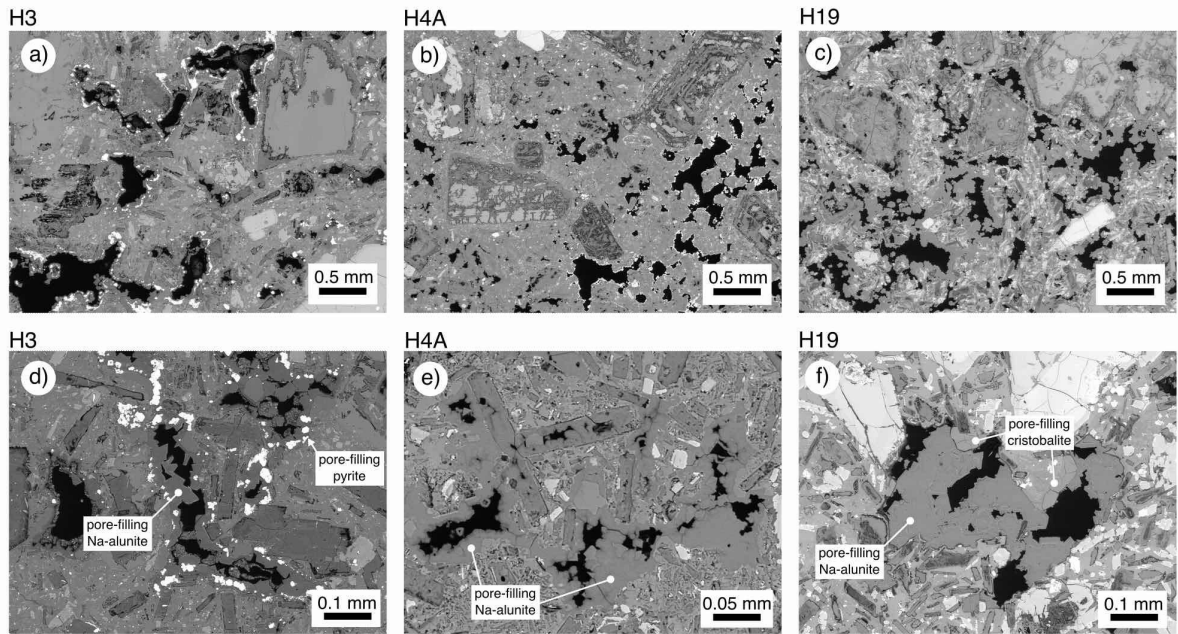


519

520 **Fig 1.** Schematic diagrams explaining the different types of lava dome heterogeneity discussed in this contribution.

521 (a) Discrete heterogeneity: the lava dome is split into discrete zones (yellow and white) that are characterised by
522 different, but uniform, physical and mechanical properties. (b) Discrete and diffuse heterogeneity: the lava dome
523 is split into two discrete zones (dark yellow and grey) characterised by different physical and mechanical properties
524 (discrete heterogeneity). In this case, the physical and mechanical properties within these zones are also
525 heterogenous (diffuse heterogeneity).

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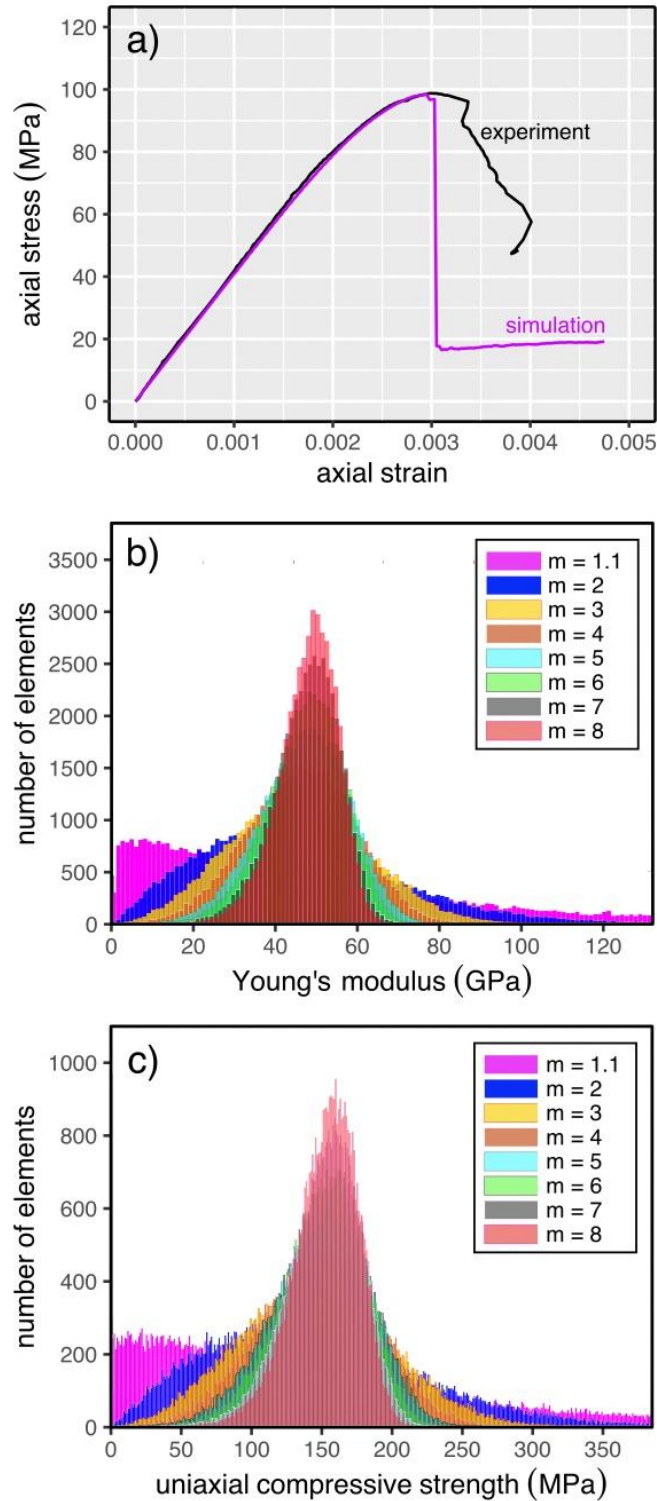
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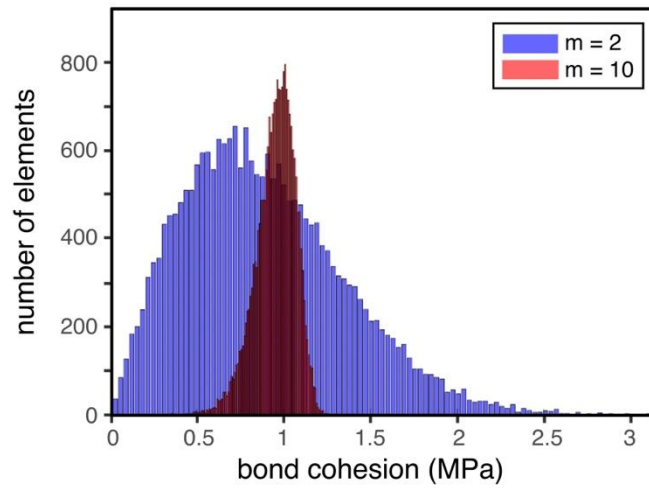
531

Fig 2. Backscattered scanning electron microscope images of the three dome rocks (H3, H4A, and H19) from La Soufrière de Guadeloupe (Eastern Caribbean) analysed in this study. Black – void space; greyscale – groundmass and crystals.



532

533 **Fig. 3.** (a) Uniaxial stress-strain curves from a laboratory experiment performed on an andesite sample (H18; data
 534 from Heap et al., 2021b) (black curve) and from a numerical experiment using Rock Failure and Process Analysis
 535 (RFPA). The parameters for the model (listed in Table 1) were modified until the modelled curve closely matched
 536 the experimental data. Panels (b) and (c) show the distribution of Young's modulus and uniaxial compressive in a
 537 numerical sample with a given homogeneity index, m (1.1, 2, 3, 4, 5, 6, 7, and 8).



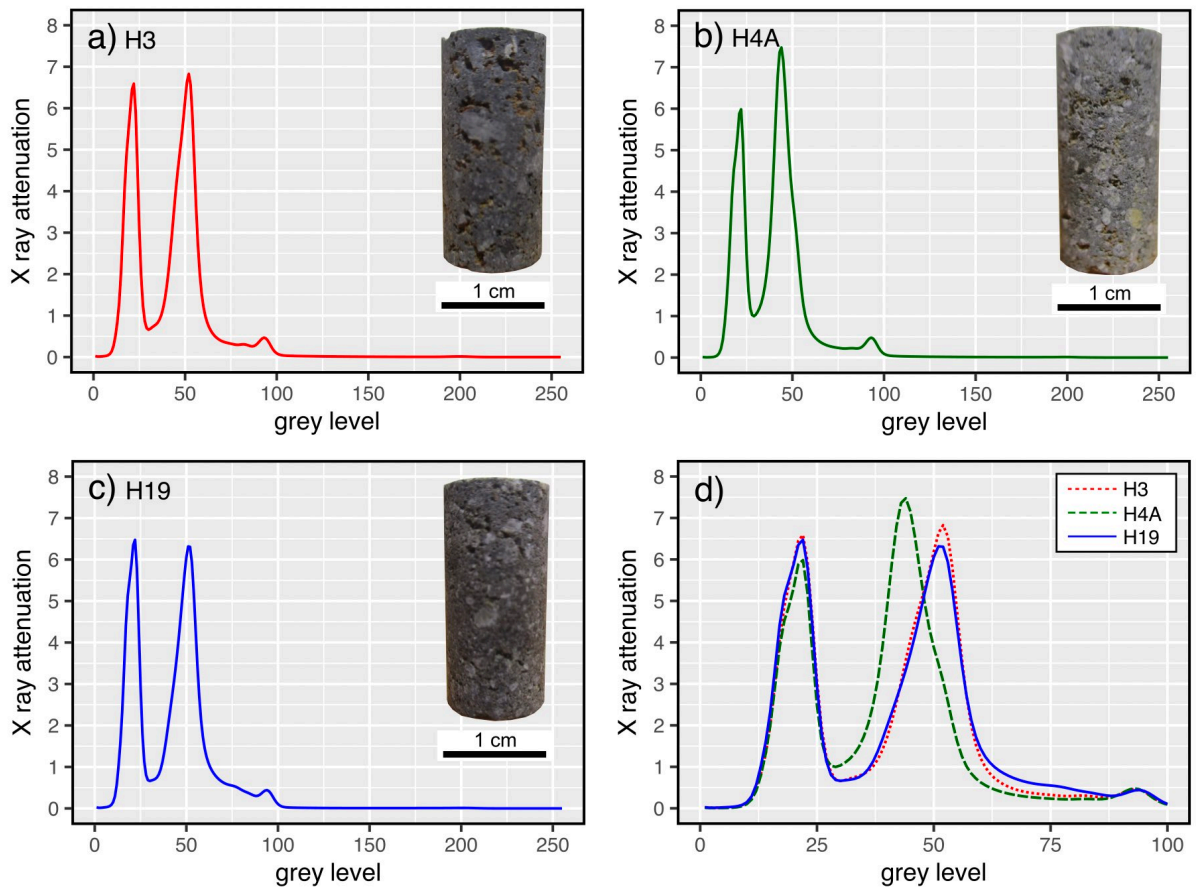
538

539 **Fig. 4.** The distribution of bond cohesion in the Particle Flow Code (PFC) domes characterised by different

540 homogeneity indices, m (the mean strength is 1 MPa in both cases). $m = 2$ (heterogeneous dome); $m = 10$

541 (homogeneous dome).

542



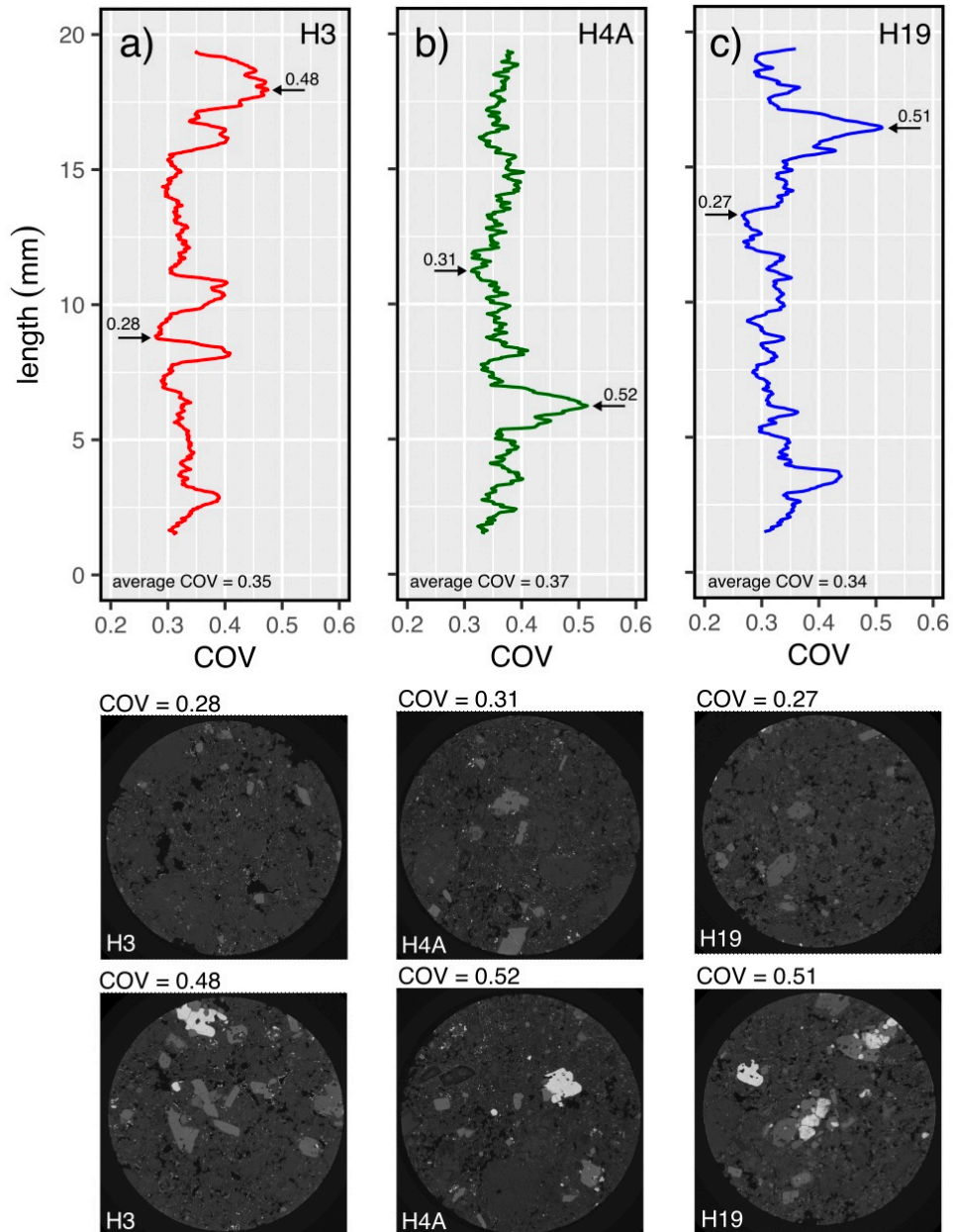
544

545 **Fig. 5.** Grey level histograms for three dome rocks from La Soufrière de Guadeloupe. Frequency is the relative

546 frequency. (a) Sample H3. (b) Sample H4A. (c) Sample H19. (d) Synopsis plot containing all the data (note that

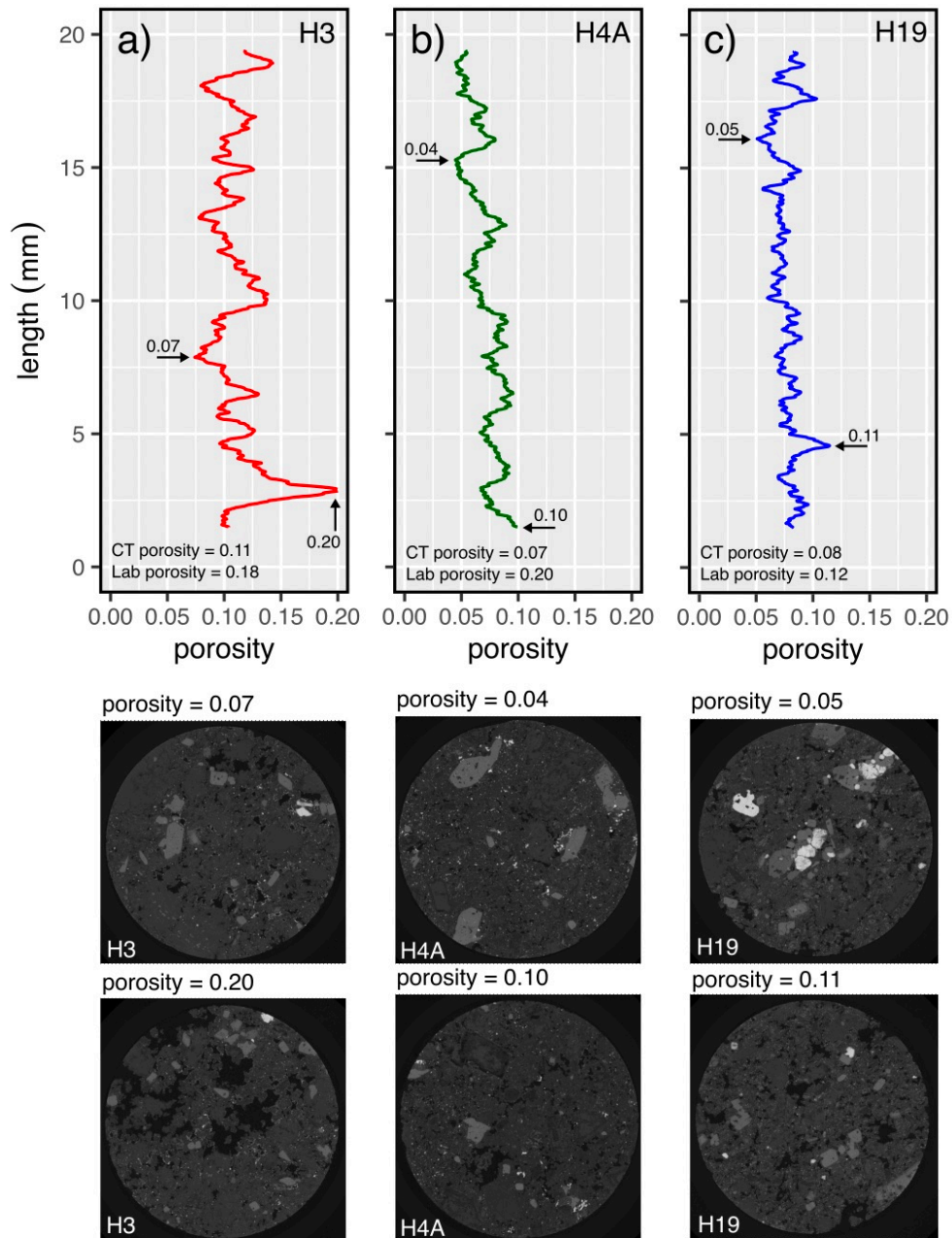
547 the scale on x-axis is not the same as for the other panels). Insets show photographs of the 10 mm-diameter samples.

548



549

550 **Fig. 6.** The variation in coefficient of variation (COV) along the length of the 20 mm-long samples. (a) Sample
 551 H3. (b) Sample H4A. (c) Sample H19. The average COV and the location of the image slice with the highest and
 552 lowest COV are indicated on the plots. Below the plots are the image slices corresponding to the highest and
 553 lowest COV for each sample.



554

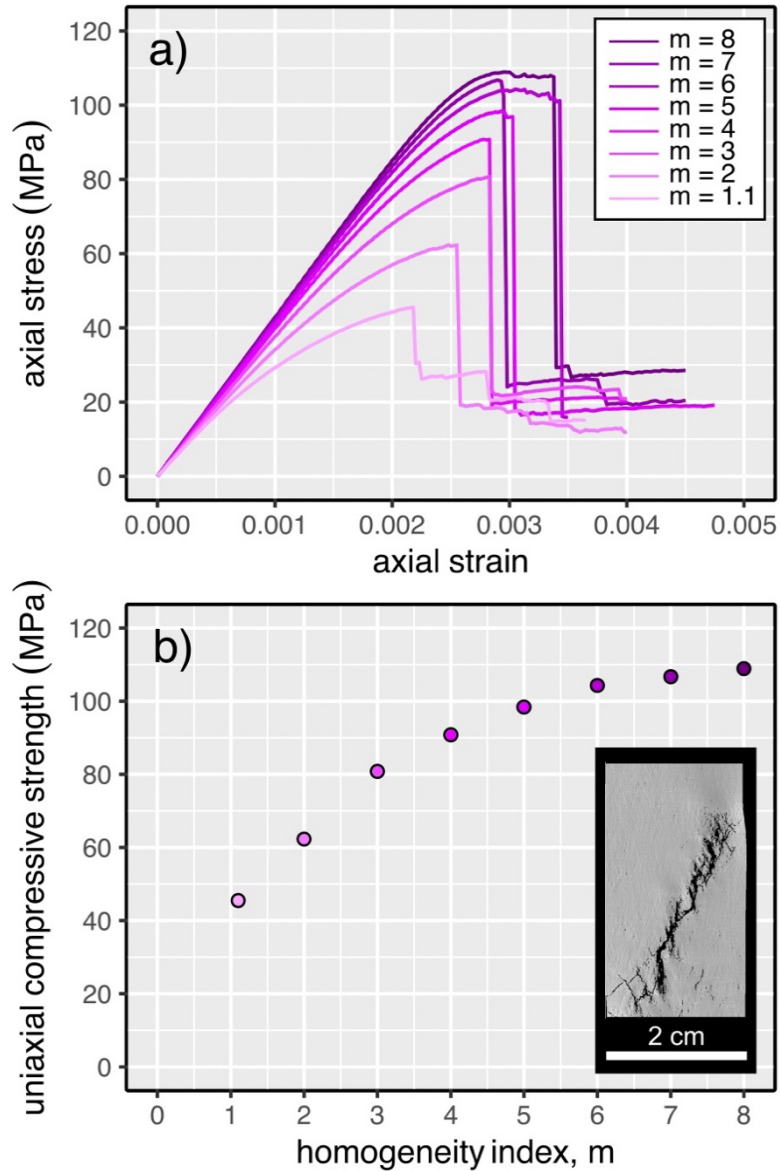
555 **Fig. 7.** The variation in porosity along the length of the 20 mm-long samples. (a) Sample H3. (b) Sample H4A. (c)

556 Sample H19. The average porosity (from the μ CT volume and the laboratory) and the location of the image slice

557 with the highest and lowest porosity are indicated on the plots. Below the plots are the image slices corresponding

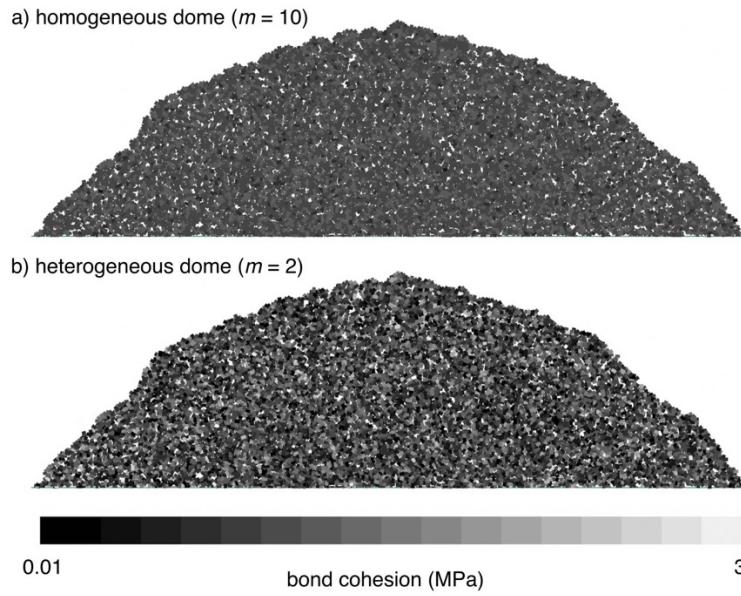
558 to the highest and lowest porosity for each sample.

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Fig. 8. Results of the 2D numerical experiments performed using Rock Failure and Process Analysis (RFPA). (a) Uniaxial stress-strain curves for numerical samples prepared with the same input parameters, but with different homogeneity indices (Table 1). The homogeneity index, m , varies from 8 (the most homogeneous sample) to 1.1 (the most heterogeneous sample). (b) Uniaxial compressive strength as a function of the homogeneity index (for the same numerical experiments shown in panel (a) (data available in Table 2). Inset shows an image of the post-failure numerical sample.

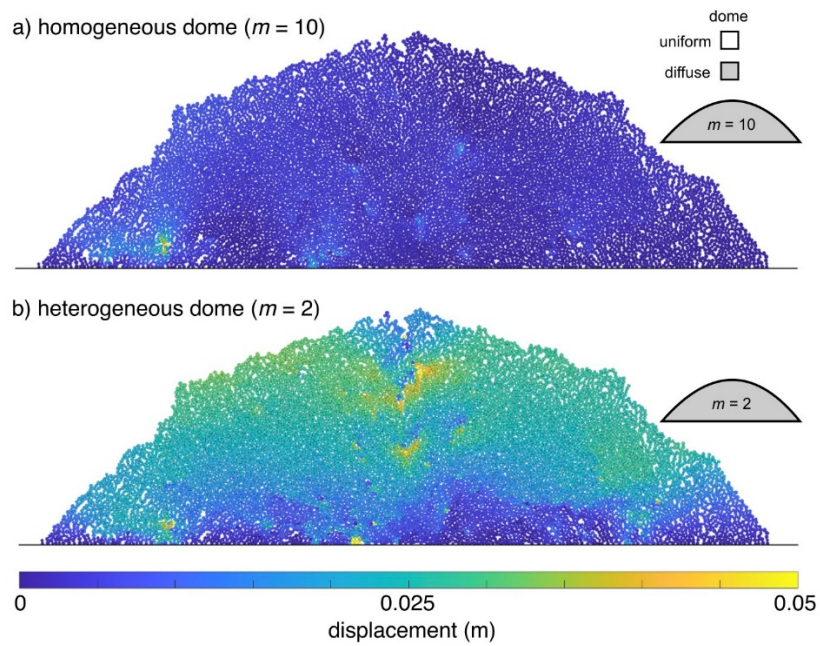


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569 **Fig. 9.** Distribution of bond cohesion in (a) a homogeneous dome ($m = 10$) and (b) a heterogeneous dome ($m = 2$).

570 Dome width ~440 m.

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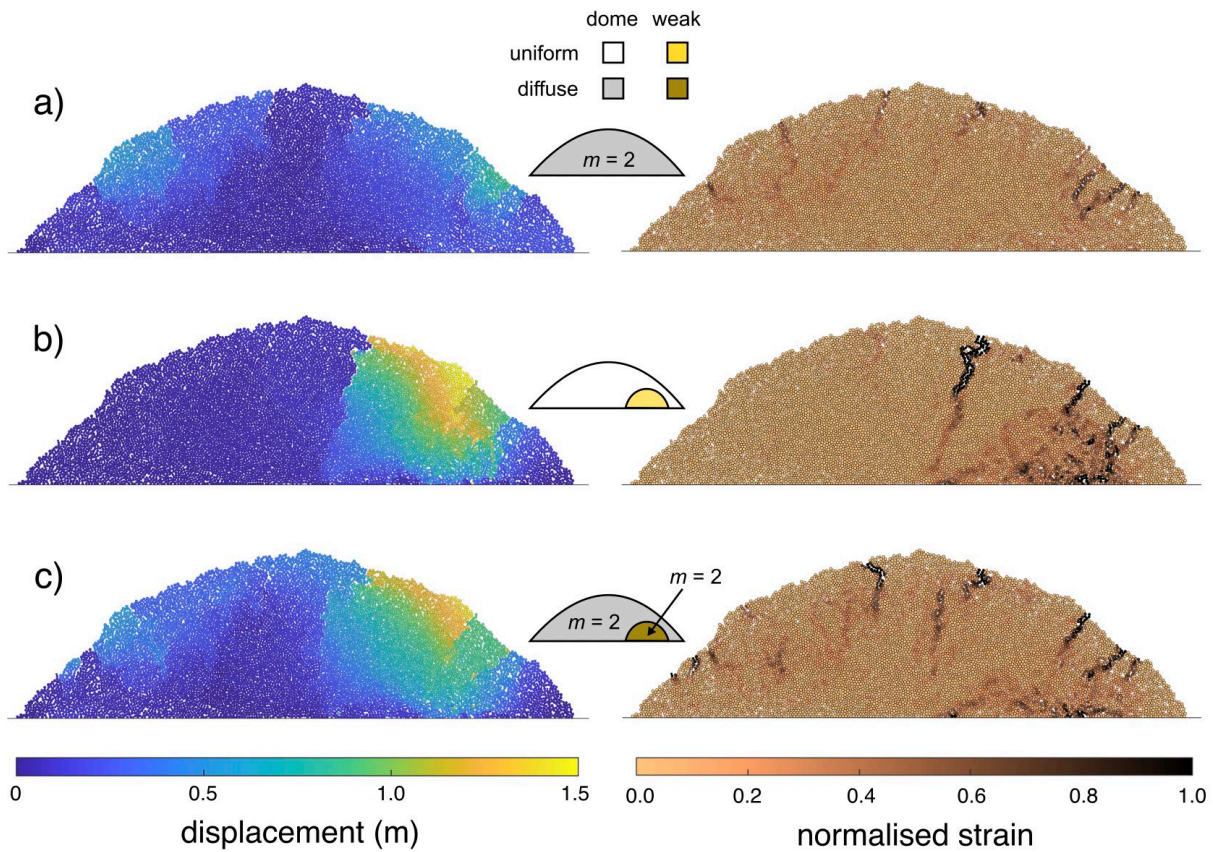
574 **Fig. 10.** Results of the 2D dome simulations (dome width ~ 440 m) performed using Particle Flow Code (PFC). (a)

575 A homogeneous dome with a homogeneity index of 10. (b) A heterogeneous dome with a homogeneity index of

576 2. The colour indicates the displacement, where blue indicates low displacement and yellow indicates high

577 displacement.

578



579

580 **Fig. 11.** Results of the 2D dome simulations (dome width ~ 440 m) performed using Particle Flow Code (PFC).

581 Left plots show displacement from 0–1.5 m (yellow indicates higher displacement) and right plots show

582 normalised strain, where darker areas indicate areas of high strain accumulation (dark brown indicates higher

583 strain). (a) Dome with diffuse heterogeneity, as in Fig. 10. This dome is calibrated to unaltered rock as described

584 by Harnett et al. (2022), with a heterogeneity index of $m = 2$, about a mean contact strength of 84.7 MPa. (b)

585 Dome with a discrete zone of weaker material (both discrete zones have uniform material properties), identical to

586 that presented by Harnett et al. (2022). (c) The combined effect of a discrete zone of weaker material *and*

587 heterogeneous material properties in all zones. The dome rock is heterogeneous with $m = 2$ about a mean strength

588 of 84.7 MPa and the weak rock is heterogeneous with $m = 2$ about a mean strength of 29 MPa. Thumbnail

589 sketches of each scenario are provided in the middle of the modelled results.

590

591 **Tables**

592

| Property | Value |
|--|--------------------------|
| Homogeneity index | 1.1, 2, 3, 4, 5, 6, 7, 8 |
| Mean uniaxial compressive strength (MPa) | 147.3 |
| Mean Young's modulus (GPa) | 46.0 |
| Poisson's ratio | 0.25 |
| Ratio of compressive to tensile strength | 10 |
| Friction angle (°) | 30 |

593

594 **Table 1.** Model input parameters for the Rock Failure and Process Analysis (RFPA) modelling.

595

| Homogeneity index, m | Uniaxial compressive strength (MPa) |
|--|--|
| 8 (most homogeneous) | 108.9 |
| 7 | 106.7 |
| 6 | 104.3 |
| 5 | 98.4 |
| 4 | 90.8 |
| 3 | 80.8 |
| 2 | 62.3 |
| 1.1 (most heterogeneous) | 45.5 |

596

597 **Table 2.** Results of the 2D numerical experiments performed using Rock Failure and Process Analysis (RFPA).

598

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